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KINETIC TEMPERATURE OF WET SURFACES

A METHOD OF CALCULATING THE AMOUNT OF ALCOHOL REQUIRED TO PREVENT ICE, AND THE DERIVATION OF THE PSYCHROMETRIC EQUATION

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A METHOD OF CALCULATING THE AMOUNT OF ALCOHOL REQUIRED TO PREVENT ICE, AND THE DERIVATION OF THE PSYCHROMETRIC EQUATION

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SUMMARY

A method is given for calculating the temperature of a surface wetted either by a pure liquid, such as water, or by a mixture, such as alcohol and water. The method is applied to the problem of protecting, by alcohol, propellers and the induction system of the engine against ice. The minimum quantity of alcohol required is calculated for a number of arbitrarily chosen conditions. The effect of evaporation of alcohol is shown by repeating the calculations for a nonvolatile fluid. The method can be applied to other problems in evaporation, for instance, to the evaporation of fuel in the induction system of the engine. The psychrometric equation, used in wet-bulb hygrometry, is deduced in its general form. The effect of kinetic heating is included in this equation.

INTRODUCTION

The subject of this note is the evaporation from a wet surface in a stream of air, the surface being unheated other than by convection of heat from the air. A familiar example

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*This report was prepared by Mr. Hardy in collaboration with the staff of the Ames Aeronautical Laboratory during a period of active participation by Mr. Hardy in the NACA icing research program.

is the wet-bulb thermometer. Evaporation is considered primarily in connection with the use of volatile organic liquids, such as alcohol, to prevent the formation of ice on parts of an aircraft, such as the propeller, or the induction system of the engine. A method is presented by which the temperature of a surface, which is wetted by a liquid consisting of one or more volatile components, may be calculated. It is possible, therefore, if the rate at which water reaches the surface is known, to calculate the minimum rate at which any particular de-icing fluid must be supplied in order to prevent freezing.

In the case of a surface wetted with water, the equation derived is the psychrometric equation in its complete form. Since the effect of kinetic heating is included, this equation may be used in the measurement of humidity, by the wet-bulb method, at high speeds. This equation is of use, also, in calculating the rate of heating required to protect a surface exposed to conditions of icing. The datum temperature, in calculating the rate of dissipation of heat from the surface by the method given in references 1 and 2, is the wet-kinetic temperature of the surface. This may be calculated from the psychrometric equation, in the form given in this report, as an alternative to the method proposed in reference 3. The use of the specific heat of wet air, as proposed in this reference, is open to the objection that it appears to require that the saturation of the air, in the vicinity of the surface, is maintained by evaporation of the droplets of water carried by the air. It appears, however, from the different approach to the problem presented in this report, that it is unnecessary to postulate a particular mechanism of evaporation.

This report is complementary to that of reference 1, since it deals primarily with the theoretical aspects of protection against ice by chemicals. The methods of calculation, though applied to the problem of icing, are applicable to problems of evaporation generally. They could be used, for instance, to calculate the rate of evaporation of fuel from a heated surface in the induction system of the engine.

SYMBOLS

The following symbols are used throughout this report:

W rate of evaporation per unit area, pounds per second, square foot

H rate of transfer of heat per unit area, British thermal unit per second, square foot

k_e coefficient of evaporation, dimensionless

k_w coefficient of evaporation of water, dimensionless

k_h coefficient of transfer of heat, dimensionless

C_f coefficient of surface friction, dimensionless

J mechanical equivalent of heat, foot-pounds per British thermal unit

g gravitational constant, feet per second squared

V_o velocity of free stream, feet per second

V_1 velocity locally at edge of boundary layer, feet per second

ρ density of air, pounds per cubic foot

ρ_e density of vapor, pounds per cubic foot

M_a molecular weight of air (= 28.96)

M_w molecular weight of water vapor $(M_w/M_a = 0.622)$

M_e molecular weight of vapor

n concentration of vapor, pounds per pound of air

e vapor pressure, millimeters of mercury

e' vapor pressure saturated air at temperature t', millimeters of mercury

p barometric pressure of air, millimeters of mercury

μ viscosity of air, pounds per second, foot

k thermal conductivity of air, British thermal units per second, square foot degree Fahrenheit per foot

c_p specific heat of air, British thermal units per pound, degree Fahrenheit

c_{p_w} specific heat of wet air, British thermal units per pound, degree Fahrenheit

D diffusivity of vapor in air, foot squared per second

L latent heat of vaporization, British thermal units per pound

t temperature, degrees Fahrenheit (unless otherwise specified)

t' wet-bulb temperature, degrees Fahrenheit (unless otherwise specified)

Tr Taylor's number, dimensionless

Pr Prandtl's number, dimensionless

Subscripts

o refers to ambient air at static temperature

1 refers to conditions at the edge of the boundary layer

s refers to the surface

w refers to water vapor

THEORY

In the analysis which follows, it is assumed that the surface wetted by the fluid is isolated thermally, so that it does not either receive or lose heat by conduction or radiation. In these circumstances, the temperature assumed by the surface is such that the heat lost by evaporation equals that gained by convection.

Rate of Evaporation

The rate of evaporation from unit area of a surface in air flowing at velocity V is given by the equation

$$W = k_e \rho V_o (n_s - n_1) \tag{1}$$

In this k_e is the coefficient of evaporation which is nondimensional, and which is related by Reynolds' analogy with the coefficient of surface friction C_f. The potential in the process of transfer is $n_s - n_1$, the difference, namely, between the concentration of vapor in equilibrium with the liquid at the surface, and that in the air outside the boundary layer. The concentration is expressed as the mass of vapor per unit mass of air; it may be expressed in terms of vapor pressures by substitution from the equation

$$n = \frac{M_e}{M_a} \frac{e}{p-e} \tag{2}$$

Equation (1) then becomes

$$W = k_e \rho V_o \frac{M_e}{M_a} \left(\frac{e_s}{p_1 - e_s} - \frac{e_1}{p_1 - e_1} \right) \tag{3}$$

in which e_s is the vapor pressure in equilibrium with the liquid at the temperature of the surface — the saturation vapor pressure , that is, in the case of a pure liquid.

Equations (1) and (3) can be applied irrespective of whether the process is one of evaporation or condensation.

The value of k_e, the coefficient of evaporation, is determined, fundamentally, by the thickness of the boundary layer and the nature of the flow, as defining the field of diffusion, and also by Taylor's number, a parameter which gives a measure of the activity of the process of diffusion. Taylor's number, Tr, is defined as

$$Tr = \frac{\mu}{D\rho}$$

in which D is the diffusivity of the vapor, and μ and ρ the viscosity and density of the gas through which the vapor is diffusing. Taylor's number is the ratio of the diffusivity of momentum, usually called kinematic viscosity, to the diffusivity of the vapor. The equation which defines diffusivity is

$$-\frac{1}{A}\frac{dm}{dT} = D\frac{d\rho_e}{dy} \qquad (4)$$

in which dm/dT is the mass of vapor diffusing in unit time across an area A, and $d\rho_e/dy$ is the concentration gradient. When applied to the diffusion of momentum, in the gas through which the vapor is diffusing, this equation becomes

$$-\frac{1}{A}\frac{d(mV)}{dT} = (D)'\frac{d(\rho V)}{dy}$$

In this equation $(D)'$ is the "self-diffusivity" of the gas, and from the kinetic theory of gases this is equal to μ/ρ. On substituting for $(D)'$, the equation becomes the normal equation which defines viscosity. Taylor's number is the ratio of $(D)'$ to D.

The conduction of heat, also, is a process of diffusion — of the energy of molecular motion. The diffusivity of heat in air is $k/\rho C_p$, and, with appropriate substitutions, equation (4) becomes the normal equation which defines the conductivity of heat. In the case of heat, the ratio of the diffusivity of momentum to that of heat is called Prandtl's number.

The value of Taylor's number, like Prandtl's number, is independent of pressure, but changes slowly with temperature. The diffusivity of a vapor in terms of its value D_{st}, at 0° C and a pressure of 1 atmosphere, is given by the equation

$$D = D_{st}\frac{p_{st}}{p}\left(\frac{t}{t_{st}}\right)^m$$

where the temperatures are in degrees absolute, and m has a value which ranges from 1.75 to 2.0.

The coefficient of evaporation k_e is related, through Reynold's analogy, to the coefficient of transfer of heat h_h as defined by the equation

$$H = k_h \rho V_o c_p (t_s - t_1 - \Delta t_1) \tag{5}$$

In this equation, Δt_1 is the increment in temperature from kinetic heating, which is discussed in a later section.

Reynolds' analogy is between heat transfer and surface friction, and equations relating the coefficients are derived in reference 4, pages 623 to 626, and 654 to 657. These equations, with substitution of coefficients and of Tr for Pr, apply also to mass transfer. For instance, when the flow is laminar

$$k_h = \frac{C_f}{2} (Pr)^{-2/3} \tag{6}$$

and

$$k_e = \frac{C_f}{2} (Tr)^{-2/3}$$

so that

$$\frac{k_e}{k_h} = \left(\frac{Pr}{Tr}\right)^{2/3} = \left(\frac{D}{kC_p}\right)^{2/3}$$

When the flow is turbulent, the equation corresponding to equation (6) is

$$\frac{1}{k_h} = \frac{2}{C_f} + 5.6 (Pr - 1) \sqrt{\frac{2}{C_f}} \tag{7}$$

This equation is approximate, but is sufficiently accurate when Pr has a value near to 1. A more accurate equation, given by Von Kármán, is

$$\frac{1}{k_h} = \frac{2}{C_f} + 5 \sqrt{\frac{2}{C_f}} \left[(Pr - 1) + \log_e \left\{ 1 + \frac{5}{6} (Pr - 1) \right\} \right]$$

The preceding equations, although derived for the flow across a flat plate and through a pipe, can be applied quite generally and without restriction to determine the value of k_e from that of k_h.

Temperature of a Wet Surface

The temperature assumed by a wet surface which is thermally isolated is such that the loss of heat by evaporation equals the gain of heat by convection from the air. The surface is isolated thermally in the sense that there is no gain or loss of heat by conduction or radiation. For such a surface the balance of flow of heat is given by the equation

$$LW = H \tag{8}$$

One volatile component.- For a surface wetted either by a pure liquid or by a solution in which one component, only, is volatile, substitution in equation (8) from equations (3) and (5) gives,

$$L\, k_e\, V_o\, \rho\, \frac{M_e}{M_a}\left(\frac{e_s}{p_1 - e_s} - \frac{e_1}{p_1 - e_1}\right) = -\, k_h\, \rho\, V_o\, C_p\, (t_s - t_1 - \Delta t_1)$$

This may be rearranged for convenience in calculating the temperature of the surface as follows:

$$t_1 - t_s + \Delta t_1 = \frac{k_e}{k_h}\,\frac{M_e}{M_a}\left(\frac{e_s}{p_1 - e_s} - \frac{e_1}{p_1 - e_1}\right)\frac{L}{C_p} \tag{9}$$

An approximate form of this equation, which may be used at low temperatures when the vapor pressure is small relative to the barometric pressure, is

$$t_1 - t_s + \Delta t_1 = \frac{k_e}{k_h}\,\frac{M_e}{M_a}\left(\frac{e_s - e_1}{p_1}\right)\frac{L}{C_p} \tag{10}$$

Two volatile components.- When the surface is wetted by a liquid consisting of two volatile components, such as alcohol and water, each evaporates independently of the other, so that the effect is additive. For a system of two components, equation (10) becomes

$$t_1 - t_s + \Delta t_1 = \frac{k_e}{k_h}\,\frac{M_e}{M_a}\left(\frac{e_s - e_1}{p_1}\right)\frac{L}{C_p} + \frac{k_w}{k_h}\,\frac{M_w}{M_a}\left(\frac{e_{s_w} - e_{1_w}}{p_1}\right)\frac{L_w}{C_p} \qquad (11)$$

The second component is indicated by the change in subscript. The vapor pressure of each component in equilibrium with the surfaces, e_s and e_{s_w}, is that appropriate to the mixture. It differs from the vapor pressure of each component in the pure state. It is important to notice that it is the composition of the liquid at the surface which is important. This, in general, will differ from the composition of the bulk of the liquid unless there is mechanical mixing, since the rates of evaporation of the components will be unequal; in fact, under certain conditions, one may be evaporating while the other is condensing on the surface.

Partially wetted surface.- Equations (9) and (10) apply to a surface which is completely wetted with liquid. Conditions may be such that less liquid is received by the surface than could be evaporated. This condition of partial wetness requires that the equations be modified by multiplying the right-hand side by the wetness fraction. The wetness fraction is determined by dividing the rate at which water is received by the surface by the rate at which it would be lost by evaporation if the surface were completely wetted. The rate of evaporation is calculated from equation (3).

The psychrometric equation.- The psychrometric equation, quoted later as equation (18), is the empirical expression of a considerable body of experimental data. It applies only at rates of air flow, past the bulbs of the thermometers, so that kinetic heating is inappreciable, and so that the field of pressure around the wet bulb is of negligible intensity as compared with the barometric pressure in the free stream. At high rates of flow, it is evident that the shape of the bulb relative to the direction of flow of air is of great importance. For instance, in the case of a cylinder across wind the temperature t_s as given by equation (9) will vary around the cylinder because of variation of Δt, and of p_1 and e_1. If the cylinder is of material of high thermal conductivity, the temperature assumed will be such as to satisfy the balance of flow of heat over the whole surface of the cylinder. The temperature of the cylinder, which will be the apparent wet bulb temperature, will be such as to satisfy the following equation

$$\oint - k_h \rho V_o C_p (t_b - t_1 - \Delta t_1) ds = \oint k_w \rho V_o \frac{M_w}{M_a}\left(\frac{e_b - e_1}{p_1 - e_1}\right) L_w \, ds$$

in which t_b is the temperature of the cylinder, and e_b the vapor pressure for saturation at this temperature.

In psychrometry, what is required is the vapor pressure of the undisturbed stream, e_o, and it is evident that this cannot be obtained by any simple process of calculation from the foregoing equation. It can be obtained, however, if the bulb of the thermometer is in the form of a flat plate, edge on to the flow of air, or is such as to satisfy the requirements of constancy both of kinetic temperature and of pressure over the surface.

In the case of a thin flat plate, both barometric and vapor pressures at the edge of the boundary layer are identical with those in the free stream. Equation (9), therefore, may be rewritten as

$$t_o - t_s + \Delta t_o = \frac{k_w}{k_h} \frac{M_w}{M_a} \left(\frac{e_s}{p_o - e_s} - \frac{e_o}{p_o - e_o} \right) \frac{L_w}{C_p} \tag{12}$$

The value of t_s, in this equation, is the temperature of the wet plate, and the equation may be used for psychrometry at high speeds.

Kinetic Heating

The effect of kinetic heating has been introduced into the equations, which give the balance of flow of heat to and from the surface, by the inclusion of Δt_1, the increase in temperature caused by kinetic heating. The temperature of the surface when the rate of convection of heat is zero, therefore, is $t_1 + \Delta t_1$.

The value of Δt_1 when the flow is laminar is given, in reference 4, by the equation

$$\Delta t_1 = \frac{V_1^2}{2gJC_p} Pr^{1/2} \tag{13}$$

When the flow is turbulent, reference 5 gives

$$\Delta t_1 = \frac{V_1^2}{2gJC_p} Pr^{1/3} \tag{14}$$

These equations presume that there is no evaporation from droplets of water which, under certain circumstances, may be present in the boundary layer, since the specific heat of dry air is used. This is consistent with the assumption, in equation (8), that evaporation occurs only at the surface. The effect of evaporation within the boundary layer will be discussed later.

Outside the boundary layer, the effect of a change of phase from vapor to liquid, is of direct importance, as it affects the value of t_1, and also the value of e_1. As air flow round a body, and its velocity changes from V_0 to V_1, the process is isentropic. The value of t_1, on the assumption of no change of phase, is

$$t_1 = t_0 + \frac{V_0^2 - V_1^2}{2gJC_p} \tag{15}$$

and the value of e_1 is

$$e_1 = e_0 \frac{p_1}{p_0}$$

Over the forward part of an airfoil, or other body, it is probable that there is no change in phase even in air which, initially, is saturated. At the other extreme, it may be assumed, in the case of an increase in velocity, that condensation occurs so rapidly that the air remains saturated. In this case, the value of t_1 is given by substituting the specific heat of wet air, C_{p_w}, in equation (15), the value of C_{p_w} being given by equation (17). The value of e_1 is the vapor pressure for saturation at t_1. Equation (17) can be modified to suit a condition of partial condensation with supersaturation, a condition intermediate between the extremes just defined.

Kinetic temperature in cloud.- When droplets of water are carried by the air stream, as is the case when an aircraft passes through cloud, the method proposed in reference 3 for calculating kinetic temperature is to use the specific heat of wet air in equations (13) and (14). The objection to this is that it appears to require evaporation from the droplets sufficient to maintain a condition of saturation throughout the boundary layer. It can be shown, however, that it is the evaporation at the surface which is of primary importance and, in the case of water, that the role of the droplets, other than in wetting the surface, is entirely neutral.

In the case of water, the diffusivity of the vapor is equal to the diffusivity of heat, as will be shown in the next section. The distribution of vapor in the boundary layer, therefore, is identical with the distribution of temperature if each is expressed in the nondimensional form

$$\frac{e - e_1}{e_s - e_1} \quad \text{and} \quad \frac{t - t_1}{t_s - t_1}$$

If the air outside the boundary layer is saturated with vapor and the surface is wet, the relation between the distribution of temperature and vapor pressure is such that a condition of saturation obtains throughout the boundary layer. It would be anticipated, therefore, that the kinetic temperature of a surface in cloud, as calculated from the specific heat of wet air, would be the same as that calculated from equation (10) for a wet surface in clear saturated air. The identity may be shown formally.

The temperature of the surface, when calculated from the specific heat of wet air, is

$$t_s = t_1 + \Delta t_{1_w} \tag{16}$$

The value of Δt_{1_w} is given by the right-hand side of equation (13) or (14) with c_{p_w} substituted for c_p, so that

$$\Delta t_{1_w} = \Delta t_1 \frac{c_p}{c_{p_w}}$$

in which

$$c_{p_w} = c_p + \frac{M_w}{M_a} \frac{L_w}{p_1} \frac{\delta e}{\delta t} \tag{17}$$

The value of $\delta e/\delta t$ is the mean for the interval of temperature $t_s - t_1$, so that

$$\frac{\delta e}{\delta t} = \frac{e_s - e_1}{t_s - t_1}$$

By substitution in equation (16)

$$t_s = t_1 + \frac{\Delta t_1}{1 + \dfrac{M_w}{M_a} \dfrac{L_w}{p_1 c_p} \left(\dfrac{e_s - e_1}{t_s - t_1}\right)}$$

When multiplied out, this gives

$$t_s = t_1 + \Delta t_1 - \frac{M_w}{M_a}\left(\frac{e_s - e_1}{p_1}\right)\frac{L_w}{C_p}$$

This differs from equation (10) only in the absence of the ratio k_e/k_h. When this ratio is unity, which requires that the diffusivities of heat and of vapor are equal, the equations are identical.

Evaluation of Taylor's Number for Water Vapor

There is some uncertainty as to the correct value of Taylor's number for water vapor, owing to uncertainty in the value of the diffusivity of the vapor. In the case of water vapor diffusing through air, the values for diffusivity obtained by different experimenters range from 0.198 to 0.252 centimeter² per second. The value may be deduced from the psychrometric equation. This is given in reference 6 as

$$e = e' - 0.000652\, p\, (t - t')(1 + 0.00102\, t') \tag{18}$$

for pressures in millimeters and temperatures in degrees centigrade. It should be compared with the accurate form given in equation (12). Transposing equation (12) and using the same symbols as in (18), with p in place of $p_o - e$, gives

$$e = e' - p\,\frac{M_a}{M_w}\,\frac{C_p}{L_w}\,\frac{k_h}{k_w}\,(t - t')$$

so that for an air temperature of 32° F

$$\frac{k_h}{k_w} = \frac{0.000652\, L_w M_w}{C_p M_a} = 1.004$$

It appears, therefore, that Taylor's number for water vapor is almost exactly equal to Prandtl's number at this temperature. The value for Prandtl's number at 32° F is 0.71.

To make Taylor's number equal to this, the diffusivity of water vapor must be 2.01×10^{-4} feet^2 per second (0.186 cm^2/sec). The value given in the International Critical Tables is 0.22 centimeter^2 per second. It is believed that the value deduced from the psychrometric equation is the more reliable, so that in evaluating equations (9) and (10), for a water-wet surface, the ratio $k_w:k_h$ should be taken as unity. The ratio changes slowly with temperature; for instance, at 0° F it is 0.996 and at 60° F, 1.007.

RESULTS OF CALCULATIONS

A number of calculations have been made in order to demonstrate the application of the preceding equations to specific problems in connection with protection of aircraft against the formation of ice. The kinetic temperature of a water-wetted surface has been calculated in order to demonstrate the effect of barometric pressure and temperature of the air. The effect of evaporation, when a volatile organic liquid is used to prevent the formation of ice, is demonstrated by calculations of the minimum amount of alcohol required for particular conditions, and comparing this with the amount required when a nonvolatile liquid, of otherwise similar properties, is used. The liquid chosen is ethyl alcohol since there are complete data for this in the International Critical Tables. A calculation has been made also for methyl alcohol to demonstrate the effect of a change in the nature of the liquid.

These calculations have been made for a section of a blade of a propeller. Calculations have been made also for the carburetor to show the mechanism whereby ice forms when the temperature of the ambient air is above freezing and the quantity of alcohol necessary to prevent the ice.

In these calculations the data for the vapor pressure of alcohol, both pure and in solution with water, have been taken from the International Critical Tables. The value of Prandtl's number has been taken as 0.71 and the value of Taylor's number for water vapor has been taken as equal to this.

Kinetic Temperatures of Propeller Blade in Wet Air

Calculations have been made of the temperatures at a point on the surface of a propeller blade, taking the local velocity as 600 feet per second and the flow as laminar. These have been made for barometric pressures of 760 millimeters and 350 millimeters, and for air temperatures (static) of 0° F and 25° F, the air being assumed to be saturated in each case. The results are given in table I. As an instance of the method, the calculation for a temperature of 0° F and pressure of 760 millimeters may be cited. For 600 feet per second, the value of Δt_1 for dry air, from equation (13) is 25.3° F, so that equation (10) for the condition specified is

$$0^\circ - t_s + 25.3^\circ = \frac{2790}{760}\,(e_s - 1.11)$$

By trial it is found that a value of 19.4° F for t_s satisfies this equation. The substantial decrease in kinetic temperature with decrease in pressure is of practical significance in the translation of the results of wind-tunnel tests, on the formation of ice, made at ground level, to the condition which will occur at high altitude when the protection from kinetic heating is less.

The kinetic temperature of a metal blade of a propeller is influenced by the conduction of heat, both along the blade and through the blade from the pressure face to the cambered face. If the blade is hollow, or is of material of low thermal conductivity, the lowest temperature will be at the position where the local velocity is greatest.

The effect of local velocity is shown by calculations for a velocity 1.4 times that of the blade of a propeller which is taken as 600 feet per second; the local velocity, therefore, is 842 feet per second. The increase in velocity chosen is less than occurs on the cambered face when the propeller is working at high incidence. The static temperature of the undisturbed air is taken as 25° F, the barometric pressure as 350 millimeters, and the air as being saturated with water vapor. At the position when the velocity reaches 842 feet per second, the pressure, calculated by Bernoulli's equation for compressible flow, is 282 millimeters. The static temperature of the air, locally, is given by equation (15) if it is assumed that the decrease in pressure is so rapid that no condensation occurs in the air. A calculation has been made on this assumption (a), and an independent calculation has been made assuming complete condensation (b).

(a) Assuming no condensation: The reduction in temperature of the air is 28.7° F so that the static temperature locally, by equation (15), is -3.7° F. The vapor pressure is $3.43 \times \frac{282}{350} = 2.76$ millimeters. If it is assumed that the flow is laminar, Δt_1 has a value of 49.6° F and equation (10) gives the value of t_s as 30.5° F. The effect of kinetic heating, therefore, is an increase in temperature, locally, of 5.5° F; at the stagnation point in clear air the increase is 30.0° F.

(b) Assuming complete condensation: The reduction in temperature of the air is given by equation (15) with the specific heat of wet air c_{p_w} substituted for c_p. The value of c_{p_w} is given by equation (17) with substitution from the equation

$$\frac{\delta e}{\delta t} = \frac{e_0 - e_1}{t_0 - t_1}$$

in which the subscript o denotes the initial value, and 1, that after expansion to 282 millimeters pressure. The values of t_1 and e_1 must be found by trial to be consistent, approximately, with the value t_1 calculated from equation (15). The reduction in temperature is found to be 13.6° F, so that the static temperature of the air locally is 11.4° F. The value of t_s, which can be calculated either from equation (16) or from equation (10), is 32.9° F.

In the foregoing calculations it is assumed that the surface is wet with water even when the temperature is below freezing. The temperature of the surface so calculated gives the datum temperature from which either the rate of heating to prevent ice, or the rate of icing, may be calculated by the methods of reference 2.

Blade Temperatures with Alcohol-Water Mixtures

Calculations have been made for the case in which ice is prevented by supplying ethyl alcohol to the blades of the propeller. The calculations have been repeated for a nonvolatile fluid having the same depressant effect on the freezing point as ethyl alcohol. The difference in the quantity of fluid required gives the excess alcohol which must be supplied in order to neutralize the refrigerating effect caused by its evaporation. A similar calculation has been made for methyl alcohol to show how volatile fluids of different composition may be compared.

In these calculations the concentration of alcohol in solution in water is such that the freezing point is depressed exactly to the temperature assumed by the blade. This may be termed the correct mixture for the particular conditions. A few calculations have been made for a surface wetted by pure alcohol to illustrate the effect of maldistribution of fluid in cooling the blade by conduction of heat from the warmer to the colder parts. The calculations were worked with the temperature in degrees centigrade because the data in the International Critical Tables are given in this scale. In many cases the calculations have been made in reverse, starting with an assumed temperature of the blade and calculating the temperature of the air required to produce this. This procedure allows a more direct comparison since the effect of evaporation is determined primarily by the temperature of the surface.

Calculations have been made for a speed of 450 feet per second assuming laminar flow, for blade temperatures of 23° F (-5° C) and also 10.6° F (-12° C), and for barometric pressures of 760 millimeters and 350 millimeters. Two calculations have been made for a speed of 250 feet per second. The results are given in table II. As an instance of the method, the calculations for the blade temperature of 23° F at 760 millimeters pressure, the first in table II, may be cited. For this condition, the concentration of alcohol required to depress the freezing point to 23° F, from the data of the International Critical Tables, is 124 grams per liter of water. The vapor pressure of alcohol in equilibrium with this mixture is 0.172 times the value for pure alcohol at the same temperature - namely, 1.44 millimeters at 23° F. The vapor pressure of water is 0.95 times the value for pure water - namely, 3.00 millimeters. Substitution in equation (11) for a speed of 450 feet per second gives

$$t_1 - 23 + 14.2 = \frac{1900}{760}(1.44 - 0) + \frac{2790}{760}(3.0 - e_{1_w})$$

The effect of evaporation is found by calculating the temperature that the blade would assume in air at 15.2° F with a nonvolatile fluid, and calculating the concentration of fluid required. The fluid is assumed to have the same depressant effect on the freezing point as ethyl alcohol. Substitution in equation (10), since in this case there is one volatile component, gives

$$15.2 - t_s + 14.2 = \frac{2790}{760} (0.96\ e_s - 2.23)$$

The vapor pressure of water in equilibrium with the mixture is 0.96 times that of pure water. From this equation, the value of t_s is 25.5° F, and the concentration of fluid of the same molecular weight as ethyl alcohol is 92 grams per liter of water. The excess of ethyl alcohol required to neutralize the refrigerating effect produced by its evaporation, therefore, is

$$\left(\frac{124 - 92}{92}\right) 100 = 35 \text{ percent}$$

The calculations show that the effect of evaporation decreases with temperature and increases with altitude. In the range of temperature in which ice is usually encountered, 15° to 32° F, the wastage of ethyl alcohol on account of its volatility is considerable. The nonvolatile fluid with which the alcohol is compared is hypothetical, but the concentrations of an actual fluid of low volatility may be calculated from those for the nonvolatile fluid, which are presented in table II. A fluid such as ethylene glycol, for instance, is practically nonvolatile, and, since it does not dissociate in solution, the amount required will be in the ratio of its molecular weight to that of the nonvolatile fluid which is the same as that of ethyl alcohol. The concentration of the nonvolatile fluid in table II, therefore, should be multiplied by 1.35 to find the equivalent concentration of ethylene glycol.

The effect of speed is shown by comparing the concentration of alcohol required at 450 feet per second, No. 2 in the table, with that required at 250 feet per second at the same temperature of the blade, No. 3, and also with that required at the same temperature of the air, No. 4. The last shows the requirements at different stations along the blade of a propeller exposed to the same conditions of icing. A comparison on the basis of the concentration of alcohol may be misleading, because it takes no account of the rate at which water is caught by the blade. The rate of catch, roughly, is proportional to the speed, so that more alcohol, actually, is required at the outer station than at the inner for the particular conditions for which the calculation was made.

A direct comparison between methyl and ethyl alcohol is given by the calculations for an air temperature of 19.2° F, Nos. 2 and 9 in the table. The weight of methyl alcohol, per 1000 grams of water, is slightly less than that of ethyl alcohol. In practice the advantage is more than nullified, probably, by maldistribution of fluid and consequent extra refrigeration of the blade as a whole. The advantage of methyl alcohol will increase as the temperature is lowered and the effect of evaporation becomes less pronounced.

So far, no account has been taken of the alcohol which is lost by evaporation. This can be calculated from equation (3), but, since the coefficient of transfer of heat must be evaluated, the rate will be specific to a particular position on the blade and particular velocity. By way of illustration, the rate has been calculated for a blade of 9-inch chord at the 5-percent-chord station for which reference 5 gives the value of $N_u/\sqrt{R}$ as 1.67. For a speed of 450 feet per second at 350 millimeters pressure, the value of k_e, therefore, is 0.0016. The rate of evaporation of ethyl alcohol from a correct alcohol-water mixture for a blade temperature of 23° F, from equation (3), is 0.53 pound per hour per square foot of area. The rate for pure ethyl alcohol, under the same conditions, is 3.13 pounds per hour per square foot.

The Carburetor

The principal feature in the icing of carburetors of modern design is the defect in kinetic heating which occurs when the velocity of the air is increased to above that of the free stream. This effect has already been discussed in connection with the temperature of the cambered surface of the blade of a propeller. In the case of the carburetor, it may be illustrated by a calculation of the temperature of the surface when the flow of air is throttled.

Calculations have been for a temperature at the entry to the carburetor of 38° F and a velocity of 938 feet per second. This temperature was chosen as being near to the limit at which ice was observed to form in the tests described in reference 7 when no fuel was supplied to the carburetor. The velocity has been calculated from Bernoulli's equation as that equivalent to the drop in pressure across the throttle, given in table II of this reference - namely, 10.7 inches of mercury. It is assumed that the air at inlet is saturated, that there is no condensation, and that the flow is turbulent. Assuming the velocity at entry to be zero, the value of t_1, from equation (15), is -34° F, and the value of Δt_1, from equation (14), is 64.8° F. The air at entry is saturated with water vapor; so, when throttled, the vapor pressure is

$$\frac{18.3}{29} \times 5.8 = 3.64 \text{ millimeters}$$

the barometric pressure at entry being assumed to be 29 inches of mercury. If the wall is wetted with water, equation (10) gives the temperature at the surface t_s as 28.7° F. Actually, at the wall there will be a gradient in temperature, so that heat conducted through the body of the carburetor will cause the temperature to be somewhat greater than that calculated. If ethyl alcohol is used to prevent freezing, the temperature at the surface for a correct mixture, from equation (11), is 26.8° F, and 72.5 grams per 1000 grams of water are required to depress the freezing point to this temperature.

At altitude, with the throttle fully open, the zone of high velocity is in the boost venturi in the Bendix-Stromberg type of carburetor. Calculations have been made for this with air at entry to the carburetor at a temperature of 34° F and pressure of 350 millimeters. A velocity of 600 feet per second was chosen, which gives a pressure at the throat of 281 millimeters. The velocity is higher, perhaps, than ordinarily would occur. On the assumption that no condensation occurs, the temperature at the surface is calculated as 29.7° F, the method of calculation being the same as that discussed in the preceding paragraph. On the assumption that condensation is complete, the temperature at the surface is 32.9° F.

These calculations are given by way of illustrating the method. Their primary use, it appears, is in calculating the rate of heating required to prevent ice by the method of references 1 and 2. The problem in protection by fluids is to distribute the fluid efficiently. In this case, the method of calculation can be used to find the minimum rate of supply required, or to compare the merits of fluids of different composition, on the assumption of perfect distribution.

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TABLE I

WET KINETIC TEMPERATURES FOR 600 FEET PER SECOND, LAMINAR FLOW

Air temperature, t_o (°F)	Pressure (mm)	Surface temperature, t_s (°F)	$\frac{\Delta t\ wet}{\Delta t\ dry}$
0	760	19.4	0.77
0	350	15.8	.62
25	760	40.1	.60
25	350	35.5	.49

TABLE II.- TEMPERATURES AND CONCENTRATION OF ALCOHOL

FOR ALCOHOL-WATER MIXTURES ON PROPELLER BLADE

No.	Velocity (fps)	Air temperature, t_o (°F)	Pressure (mm)	Surface temperature, t_s (°F)	Concentration of fluid (grams/1000g water)	Remarks	Excess alcohol required (percent)
1	450 450	15.2 15.2	760 760	23.0 25.5	124 92	For ethyl alcohol For nonvolatile fluid	35
2	450 450	19.2 19.2	350 350	23.0 26.3	124 70	For ethyl alcohol For nonvolatile fluid	77
3	250 250	24.2 24.2	350 350	23.0 26.6	124 75	For ethyl alcohol For nonvolatile fluid	66
4	250 250	19.2 19.2	350 350	18.0 22.1	184 135	For ethyl alcohol For nonvolatile fluid	36
5	450 450	15.2 19.2	760 350	14.7 11.5	---- ----	Pure ethyl alcohol in air saturated with water vapor	----
6	450 450	1.6 1.6	760 760	10.6 13.3	270 239	For ethyl alcohol For nonvolatile fluid	13
7	450 450	5.5 5.5	350 350	10.6 15.8	270 212	For ethyl alcohol For nonvolatile fluid	27
8	450 450	21.9 21.9	350 350	23.0 29.1	86 27	For methyl alcohol For nonvolatile fluid	218
9	450 450	19.2 19.2	350 350	19.9 26.8	113 49	For methyl alcohol For nonvolatile fluid	132